## Lecture 4

## Cascades and hybrid systems

A typical cascade is shown in Figure 1, where, in each stage, an attempt is made to bring two or more process streams of different phase state and composition into intimate contact to promote rapid mass and heat transfer, so as to approach physical equilibrium. The resulting phases, whose compositions and temperatures are now closer to, or at, equilibrium, are then separated and each is sent to another stage in the cascade, or withdrawn as a product. Although equilibrium conditions may not be achieved in each stage, it is common to design and analyze cascades using equilibrium-stage models. Alternatively, in the case of membrane separations, where phase equilibrium is not a consideration and mass-transfer rates through the membrane determine the separation, cascades of membranes can enable separations that cannot be achieved by contact of the feed mixture with a single-membrane separator.

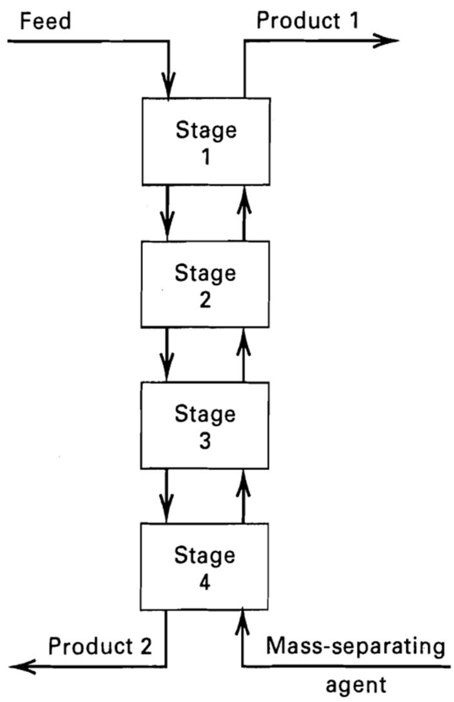

Figure 1 - Cascade of contacting stages

Cascades can be configured in many ways, as shown by the examples in Figure 2, where stages are represented by either boxes, as in Figure 1, or as horizontal lines in Figure 2d,e. Depending on the mechanical design of the stages, cascades may be arranged vertically or horizontally. The feed to be separated is designated by F; the mass-separating agent, if used, is designated by S; and products are designated by $P_i$.

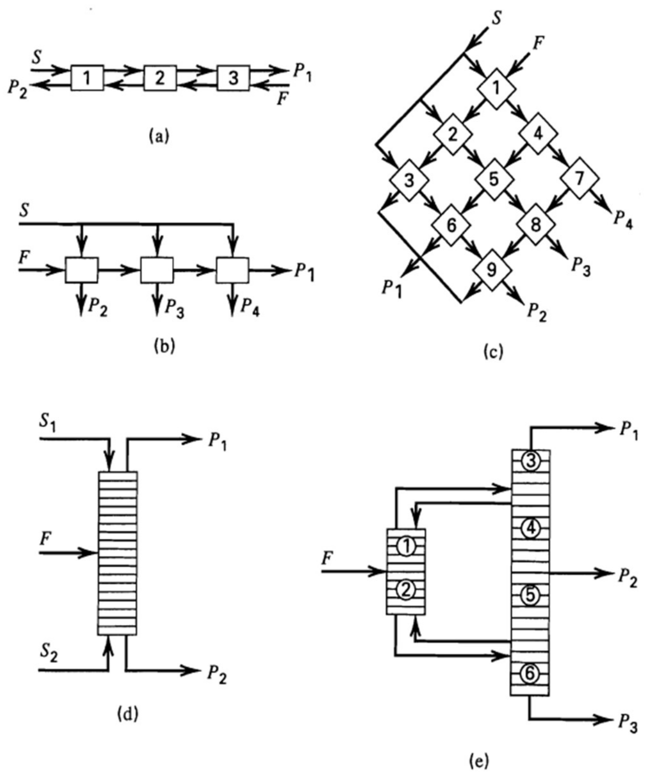


Figure 2 - Examples of cascade configurations: (a) countercurrent cascade; (b) crosscurrent cascade; (c) two-dimensional, diamond cascade; (d) two-section, countercurrent cascade; (e) interlinked system of countercurrent cascades.

In the countercurrent cascade, shown in Figures 1 and 2a, the two phases flow countercurrently to each other between stages. As will be shown in examples, this configuration is very efficient and is widely used for absorption, stripping, liquid-liquid extraction, leaching, and washing. The crosscurrent cascade, shown in Figure 2b, is, in most cases, not as efficient as the countercurrent cascade, but it is easier to apply in a batchwise manner. It differs from the countercurrent cascade in that the solvent is divided into portions fed individually to each stage.

A complex diamond variation of the crosscurrent cascade is shown in Figure 2c. Unlike the two former cascades, which are linear or one-dimensional, the diamond configuration is two-dimensional. One application is to batch crystallization. Feed F is separated in stage 1 into crystals, which pass to stage 2, and mother liquor, which passes to stage 4. In each of the other stages, partial crystallization or recrystallization occurs by processing crystals, mother liquor, or combinations of the two. Final products are purified crystals and impurity-bearing mother liquors.

The first three cascades in Figure 2 consist of single sections with streams entering and leaving only from the ends. Such cascades are used to recover components from a feed stream and are not generally useful for making a sharp separation between two selected feed components, called key components. To do this, it is best to provide a cascade consisting of two sections. The countercurrent cascade of Figure 2d is often used. It consists of one section above the feed and one below. If two solvents are used, where S1 selectively dissolves certain components of the feed, while S2 is more selective for the other components, the process, referred to as fractional liquid-liquid extraction, achieves a sharp separation. If S is a liquid absorbent and Sz is a vapor stripping agent, added to the cascade, as shown, or produced internally by condensation heat transfer at the top to give liquid reflux, and boiling heat transfer at the bottom to give vapor boilup, the process is simple distillation, for which a sharp split between two key components can be achieved if a reasonably high relative volatility exists between the two key components and if reflux, boilup, and the number of stages are sufficient.

Figure 2e shows an interlinked system of two distillation columns containing six countercurrent cascade sections. Reflux and boilup for the first column are provided by the second column. This system is capable of taking a ternary (three-component) feed, F, and producing three relatively pure products, P1, P2, and P3. In this chapter, algebraic equations are developed for modeling idealized cascades to illustrate, quantitatively, their capabilities and advantages. First, a simple countercurrent, single-section cascade for a solid-liquid leaching and/or washing process is considered. Then, cocurrent, crosscurrent, and countercurrent single-section cascades, based on simplified component distribution coefficients, are compared for a liquid-liquid extraction process. A two-section, countercurrent cascade is subsequently developed for a vapor-liquid distillation operation. Finally, membrane cascades are described. In the first three cases, a set of linear algebraic equations is reduced to a single relation for estimating the extent of separation as a function of the number of stages in the cascade, the separation factor, and the flow ratio of the mass- or energy-separating agent to the feed.